

**Generalized Sobel test with robust standard errors for statistical**

**validation of serial mediation effects on learning research**

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$$Z\_{sobel}=\frac{ab}{\sqrt{b\_{}^{2}SE\_{a}^{2}+a\_{}^{2}SE\_{b}^{2}}}$$

**Sobel test equation**

$$Z\_{exact}=\frac{ab}{\sqrt{b\_{}^{2}SE\_{a}^{2}+a\_{}^{2}SE\_{b}^{2}+SE\_{a}^{2}SE\_{b}^{2}}}$$

**Aroian test equation**

$$Z\_{unbiased}=\frac{ab}{\sqrt{b\_{}^{2}SE\_{a}^{2}+a\_{}^{2}SE\_{b}^{2}-SE\_{a}^{2}SE\_{b}^{2}}}$$

**Goodman test equation**

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* Sobel test delta test

$$SE\_{δ}^{2}=\left(\prod\_{i=1}^{n}a\_{i}\right)^{2}\left(\sum\_{i=1}^{n}\frac{SE\_{a\_{i}}^{2}}{a\_{i}^{2}}\right)$$

* Arorian exact test

$$SE\_{exact}^{2}=\left(\prod\_{i=1}^{n}a\_{i}\right)^{2} \left[\sum\_{m=1}^{n}\sum\_{1\leq i\_{1}<i\_{2}<…<i\_{m}\leq n}^{}\left(\sum\_{i=1}^{n}\frac{SE\_{a\_{ik}}^{2}}{a\_{i}^{2}}\right)\right] $$

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* Goodman Unbiased test

$$SE\_{unbiased}^{2}=\sum\_{i=1}^{n}\left(SE\_{a\_{i}}^{2}\prod\_{j\ne i}^{}a\_{i}^{2}\right)^{}- \left[\sum\_{i=1}^{n}\sum\_{j>i}^{}\left(SE\_{a\_{i}}^{2}SE\_{a\_{j}}^{2}\prod\_{k\ne i, j}^{}a\_{k}^{2}\right)\right] $$

**Abstract**

A serial mediation effect refers to a mediation

The purpose of this study is to derive the generalized results through mathematical

effect that emerges through multiple stages

within a mediation model, such as 'X->M1-

>M2 ->...-> Y'. Recently, high-performance PCs have enabled the application of the Bootstrap technique to obtain confidence intervals and determine statistical significance. However, when raw data is unavailable or when attempting to calculate mediation effects from other studies, the Sobel test can be utilized. Yet, the calculation of the Sobel test’s standard error becomes complex when there are more than three coefficients in a path, highlighting the need for a dedicated computational tool. Additionally, when the sample size is small, adjustments for normal distribution and heteroscedasticity issues are required. Consequently, this study proposes a method for calculating the standard error of the generalized Sobel test and introduces a calculator that implements this method.

**Research Question**

1. What are the sobel, ecact, and unbiased standard error formulas for more than three coefficients of the indirect effect?
2. What is the formula for the generalized standard error, and what is the description of the heteroscedasticity?
3. how is the comparison between the generalized Sobel test and the bootstrap test?

**Objective**

The calculation methods for the standard error of the Sobel test include Sobel's (1982) Delta method, the exact test method popularized by Baron and Kenny (1986) based on Aroian (1944),

and Goodman's (1960) unbiased test

modeling to calculate the standard error when the number of coefficients on the path increases to investigate the serial mediating effect of the Sobel test method. In addition, the indirect effects of the Sobel test often show heteroscedasticity without following a normal distribution; a strong standard error must be applied to solve these problems.The ultimate goal is to achieve the same level of statistical significance as obtained through the bootstrap method using raw data without the need for raw data or high-performance PCs.

*Aroian exact method standard error*

$$SE\_{exact}= \sqrt{(a\_{2}^{}a\_{3}^{})^{2}SE\_{a\_{1}}^{2}+(a\_{1}^{}a\_{3}^{})^{2}SE\_{a\_{2}}^{2}+(a\_{1}^{}a\_{2}^{})^{2}SE\_{a\_{3}}^{2}}$$

 $+\left(a\_{3}^{2}  SE\_{a\_{1}}^{2}  SE\_{a\_{2}}^{2}\right) +\left(a\_{2}^{2}   SE\_{a\_{1}}^{2}  SE\_{a\_{3}}^{2}\right)+(a\_{1}^{2}SE\_{a\_{2}}^{2}SE\_{a\_{3}}^{2})$

**Methods**

This study explores and mathematically models the issue of having multiple path coefficients in a model by conducting a literature review to identify possible solutions. Furthermore, the standard error of the generalized Sobel test was inductively proven. In addition, a comparative analysis was conducted to verify whether there are any differences in statistical significance between the results obtained using the original data through the bootstrap method and those analyzed using the generalized Sobel test. The heteroscedasticity problem was also mitigated through White's (1960) robust standard error correction, which uses the sample size and regression coefficients for adjustment. To facilitate researchers' calculations, a calculator was developed using a Shiny app, addressing the difficulties in computation.

**Findings**

Taylor et al. (2008) proposed a calculation method for cases where there are three path coefficients. Similarly, this study also models this mathematically and generalizes it using an inductive approach. The generalization through mathematical modeling is as follows:

𝑖𝑛𝑑 𝑒𝑓𝑓𝑒𝑐𝑡 = 𝑎1 × 𝑎2 × 𝑎3

This has been implemented as a Shiny app, and

*Sobel delta method standard error*

$$SE\_{ind}= \sqrt{(a\_{2}^{}a\_{3}^{})^{2}SE\_{a\_{1}}^{2}+(a\_{1}^{}a\_{3}^{})^{2}SE\_{a\_{2}}^{2}+(a\_{1}^{}a\_{2}^{})^{2}SE\_{a\_{3}}^{2}}$$

the link to the developed web app is as follows:**https://parkjoonghee.shinyapps.io/sobel/**

Here's a model for comparing Bootstrap and Sobel test results.



Figure 2. Online Learning Satisfaction Model(Park, 2024)



Although there are slight differences between the bootstrap results and the calculations of the generalized Sobel test, the results of statistical significance were consistent in both analyses. In other words, when the generalized Sobel test applies robust standard errors, it mitigates issues of non-normality and heteroscedasticity, making it possible to check the statistical significance of mediation effects.

**Conclusion**

Firstly, the generalized Sobel test presents a mathematical model that can determine the statistical significance of indirect effects even when the number of path coefficients increases. This is significant as it provides a method for testing the statistical significance of serial mediation effects. Secondly, the Shiny app has been developed to

method.

Figure 1. sobel test site app



simplify the calculation process, making it easy for anyone to calculate serial mediation effects. Thirdly, although there are slight differences in the standard

*Goodman Unbiased method standard error*

$$SE\_{unbiased}= \sqrt{(a\_{2}^{}a\_{3}^{})^{2}SE\_{a\_{1}}^{2}+(a\_{1}^{}a\_{3}^{})^{2}SE\_{a\_{2}}^{2}+(a\_{1}^{}a\_{2}^{})^{2}SE\_{a\_{3}}^{2}}$$

 $-\left(a\_{3}^{2}SE\_{a\_{1}}^{2}SE\_{a\_{2}}^{2}\right) -\left(a\_{2}^{2}SE\_{a\_{1}}^{2}SE\_{a\_{3}}^{2}\right)-\left(a\_{1}^{2}SE\_{a\_{2}}^{2}SE\_{a\_{3}}^{2}\right)+(SE\_{a\_{1}}^{2}SE\_{a\_{2}}^{2}SE\_{a\_{3}}^{2})$

errors and confidence intervals between the results

obtained from the bootstrap method and the generalized Sobel test, their statistical significance is consistent. Furthermore, the use of robust standard errors has added precision to the statistical significance testing, which is also meaningful.

Reference

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